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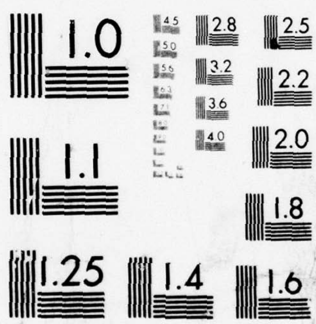

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Technical Memorandum-to-File No. 53

ARRAY STEERING IN A LAYERED WAVEGUIDE

by

C. S. Clay

October 3, 1960

Columbia University
Hudson Laboratories
Dobbs Ferry, N. Y.

Contract N6-ONR-27135

and

$$G(\omega)p_m p_n = \bar{p}_m \bar{p}_n / \Delta\omega_{mn} \text{ for } (\omega_o - \Delta\omega_{mn}/2) \leq \omega \leq (\omega_o + \Delta\omega_{mn}/2)$$

or

$$G(\omega)p_m p_n = 0 \text{ for } \omega \text{ otherwise.}$$

K_m, U_m, and r_m are evaluated at $\omega = \omega_o$ in Eq. (4.4). The $n = m$ terms are the covariance of the same mode observed at the two detectors. With the range of time delay $|\tau_1 - \tau_2|$ limited to about the travel time between the detectors $|(x_1 - x_2)/U_m|$, the most important terms in the summation are

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6 ARRAY STEERING IN A LAYERED WAVEGUIDE *†‡

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Columbia University, Hudson Laboratories
Dobbs Ferry, New York

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ABSTRACT

The usual way to determine the direction of a radiating source using an array is to steer the array for maximum output. The steering can be done by time delays or by mechanically turning the array. The output of the array can be expressed as the sum of the time average of the products of the pressure $\langle P_m P_n \rangle$ observed at detectors m and n . The value of $\langle P_m(\tau_m) P_n(\tau_n) \rangle$ can be maximized by a proper choice of time delays τ_m, τ_n . This procedure is straightforward in an infinite homogeneous medium. If the medium is a layered waveguide, there are many more possibilities for submaxima of the $P_m P_n$ terms.

The normal-mode solution of the radiation field of a band limited noise point source in a layered waveguide was given by the author in J. Acoust. Soc. Am. 31, 1473-1479 (1959). The value of $\langle P_m(\tau_m) P_n(\tau_n) \rangle$ as a function of time delay is compared with the value of $P_n P_m$ obtained with mechanical steering. For these calculations the noise source is assumed to have a bandwidth of 1/15 of the center frequency, and the depth of the water is assumed to be about 2λ over a thick layer of unconsolidated sediment. The number of maxima of the $\langle P_m(\tau_m) P_n(\tau_n) \rangle$ is related to the number of modes propagating in the waveguide if the steering is done with time delays. Mechanical steering yields one maximum that corresponds to the source direction.

1. Introduction

The use of an array to determine the direction of a radiating source has been studied with many array designs, processing techniques, and signal-to-noise conditions.[1-7] The usual way to determine the direction of the source is to adjust the phases of the signals observed at each detector so that the time average of the square of the sums of the outputs is a maximum. The phase adjustments can be made by mechanically turning the array or by time delays. Assumptions common to all the studies are that the array is in a homogeneous medium, and, in the absence of noise, that the signal observed at one detector is essentially the same as the signal observed at any other detector. If the medium is not homogeneous but is a layered waveguide, a number of things can happen. The medium is dispersive, i.e., different frequencies travel with different group and phase velocities. Depending on the number of solutions allowed by the boundary conditions, a particular frequency can have several different group and phase velocities. In addition to these difficulties, the phase and group velocities are not equal. Thus the signal changes as it travels in the guide, and characteristics of the received signal are largely due to the waveguide.[8-9]

In this paper we re-examine the array steering problem with both the array and source in a layered waveguide. The solution of the pressure field due to a point source that radiates band limited noise in a layered waveguide has been given in terms of normal modes.[10] This pressure field is used with the usual procedure of array calculations in the following sections. A numerical example with a two-element array and a shallow-water waveguide is given. I should remark here that this development is mainly concerned with waveguides that are a few acoustic wavelengths thick and with very large source-receiver distances. The source is driven by band limited noise, and, in the numerical example, the bandwidth is one fifteenth of the center frequency.

2. Propagation in a Layered Waveguide

Tolstoy has given the theory of propagation of acoustic waves due to a simple harmonic point source in a layered waveguide in detail.[11-13] It is sufficient here to summarize his results.

All energy from a source in a uniform layer that is totally reflected at the boundaries of the layer is trapped in the layer. The trapped energy spreads cylindrically and has many reflections from the top of, within, and at the bottom of the waveguide. The most convenient way to describe the pressure field at a large distance is to use the normal-mode formulation of the problem. We assume a horizontally stratified medium with a free surface on the top. This is basically the shallow-water problem with water over layers of sediment extending to basement.

The simple harmonic source is at depth d in the first layer. The detector is at depth z in the first layer and at range R from the source. The long-range solution for the acoustical pressure P' is the following:

(2.1)

$$P' = i\rho_1 A R^{-1/2} e^{i\omega t} \sum_{m=1}^{M} p_m \sin r_{1m} z \sin r_{1m} d \times \exp\left[-i\left(K_m R + \frac{\pi}{4}\right)\right] ,$$

in which

ω = the angular frequency,

K_m = the horizontal component of the wave number for the mth mode,

$K_m = \omega / c_m$,

c_m = the phase velocity of the mth mode,

r_{im} = the vertical component of the wave number in the ith layer,

$r_{im} = (\omega^2/a_i^2 - K_m^2)^{1/2}$,

p_m = the mode excitation for mth mode,

a_i = velocity of sound in the ith layer,

and

ρ_i = density in the ith layer.

It is difficult to express K_m, r_{im}, and p_m as functions of ω in terms of the parameters of the waveguide. For a particular waveguide, numerical solutions are computed. There are many papers in the literature that give the theory and techniques.[11-16] For our purpose, the most important thing is that simple functions of ω can be used to approximate the numerical values of $r_{im}(\omega)$, $K_m(\omega)$, and $p_m(\omega)$.

The extension of the simple harmonic source to the band limited noise source is obtained from Eq. (2.1) by using the Fourier integral expression for the noise source:

(2.2)

$$P(t)=\int_{-\infty}^{\infty}\left\{g(\omega)iAR^{-1/2}e^{i\omega t}\sum_{m=1}^{M}p_m \sin r_{1m}z \sin r_{1m}d \times \exp[i(K_m R+\tfrac{\pi}{4}]\right\}d\omega \ .$$

The source and receiver filter characteristics are included in the function $g(\omega)$.

3. Array of Detectors

The usual way to process the signals from an array of detectors is to add the signals and take the time average of the square of the sum. The direction to source can be determined by measuring the output of the array as a function of steering direction. The objective of steering is to match the phases of the signal from each detector so that all the signals are the same. Array steering can be done by turning the array or by placing the proper time delay in each signal channel.

Let us assume that we have an array of detectors 1, 2, ... N . The detectors measure acoustical pressures $P_1(t)$, $P_2(t)$, ... $P_N(t)$. Time delays are placed in each signal channel so the signals observed become (without change of notation) $P_1(t-\tau_1)$, $P_2(t-\tau_2)$, ... $P_N(t-\tau_N)$. The output of an additive array is the sum of the individual signals. The time average of the square of the array output is the following:

(3.1)
$$F = \sum_{1}^{N} \sum_{1}^{N} \left\langle P_m(t-\tau_m) \; , \; P_n^*(t-\tau_n) \right\rangle \, ,$$

in which P^* is the complex conjugate of P and

(3.2)
$$\left\langle P_m P_n^* \right\rangle \equiv \underset{\lim T\to\infty}{\text{Real}} \frac{1}{2T} \int_{-T}^{T} P_m(t-\tau_m) P_n^*(t-\tau_m) dt \, .$$

Equation (3.1) can be separated into two sets of terms, the squared terms $\left\langle P_m P_m^* \right\rangle$ and the cross terms $\left\langle P_m P_n^* \right\rangle$. The squared terms are independent of steering and time delay for a source of stationary noise. The directional information is obtained from cross terms. Upon separation of the terms in the summation, Eq. (3.1) becomes

(3.3)
$$F = \sum_{1}^{N} \left\langle P_m^2 \right\rangle + \sum_{\substack{1 \\ n\neq m}}^{N} \sum_{1}^{N} E_{mn}(\tau_m - \tau_n) \, ,$$

in which

(3.4)
$$E_{mn}(\tau_m - \tau_n) \equiv \left\langle P_m(t-\tau_n) \cdot P_n^*(t-\tau_n) \right\rangle \, .$$

The cross terms are also the covariances of pairs of signals with time delays as defined by Eq. (3.4).[17] The balance of this paper will discuss a single covariance E_{12}. This simplification is mainly one of notation. It does not limit the results to a two-element array because all of the cross terms in Eq. (3.3) are similar.

4. Covariance of Two Signals

The coordinate axis is shown in Fig. 1. The detectors

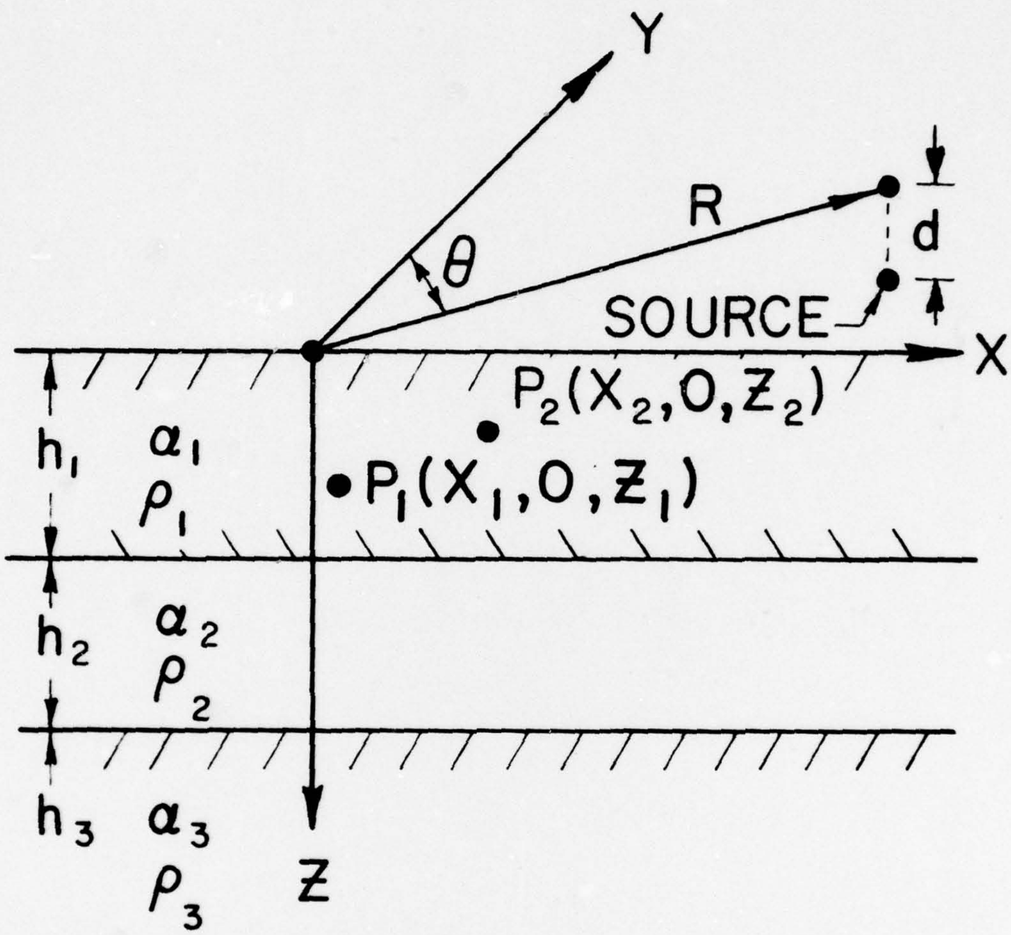

Fig 1

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are in the xz plane at (x_1, z_1) and (x_2, z_2). The source is at depth d, distance R, and angle θ as measured from the y axis. The distance of the source is presumed to be large relative to the thickness of the waveguide and the separation between the detectors.

The covariance is obtained from Eq. (2.2) and Eq. (3.2). The product of the integrals can be transformed to a multiple integral by changing the variable of integration from ω to ω'. The integration over t and application of the limit as T tends to infinity indicates that the expression vanishes for $\omega \neq \omega'$.[17] Thus one obtains the following:

(4.1)

$$F_{12}(\mathcal{T}) = \text{Real} \frac{A^2 \rho^2}{R_1^{1/2} R_2^{1/2}} \sum_{m,n} \int_{-\infty}^{\infty} G(\omega) p_m p_n \sin r_{1m} z_1 \sin r_{1n} z_2$$

$$\times \sin r_{1m} d \sin r_{1n} d \exp \left\{ i[K_m R_1 - K_n R_2 - \omega(\mathcal{T}_1 - \mathcal{T}_2)] \right\} d\omega ,$$

in which

$$G(\omega) \equiv g\, g^*$$

$$R_1 \equiv R^2 - 2Rx_1 \sin\theta + x_1^2$$

$$R_2 \equiv R^2 - 2Rx_2 \sin\theta + x_2^2$$

and

(4.2)
$$\left.\begin{aligned} R_1 &\approx R - x_1 \sin\theta \\ R_2 &\approx R - x_2 \sin\theta \end{aligned}\right\} \text{ for } \begin{aligned} R &\gg x_1 \\ R &\gg x_2 \end{aligned} .$$

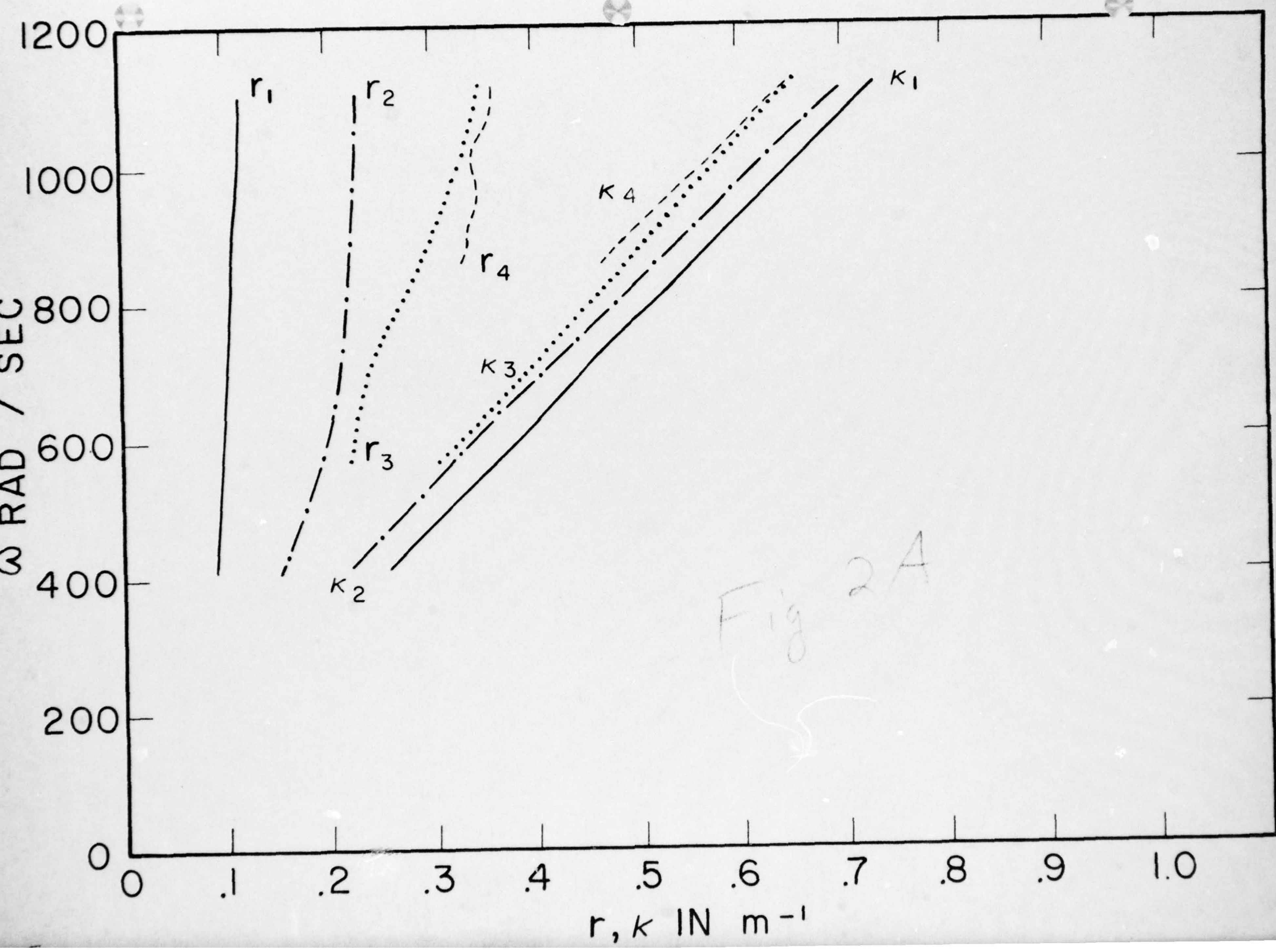
Equation (4.1) can be evaluated if the dependences of G, P_m, r_m, and K_m upon ω are known. Numerical values of K_m and r_m as functions of ω are shown in Fig. 2A, and numerical values of $p_m(\omega)$ are shown in Fig. 2B. The calculations were made by Tolstoy for the shallow-water section near Fire Island.[18] For first approximation, K_m and r_m may be expanded as linear functions of ω. Using the following definition of the group velocity

$$U_m = \left.\frac{d\omega}{dK_m}\right|_{\omega_o} ,$$

we have

(4.3)
$$\left.\begin{aligned} K_m &\approx (\omega - a_m)/U_m \\ r_m &\approx f_m\omega - b_m \end{aligned}\right\} \text{ for } \omega \geq 0 .$$

The approximate expressions for K_m and r_m can be substituted into Eq. (4.1). If the bandwidth of the filter function $G(\omega)$ is narrow, a simple function can be used for p_m and the integrals evaluated. Wide bands can be treated by piecewise integration of several narrow bands. One must be careful in this integration since U_m and K_m are positive for outgoing waves. ω is restricted to positive values in



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r, k IN m⁻¹
r1
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Fig 2A

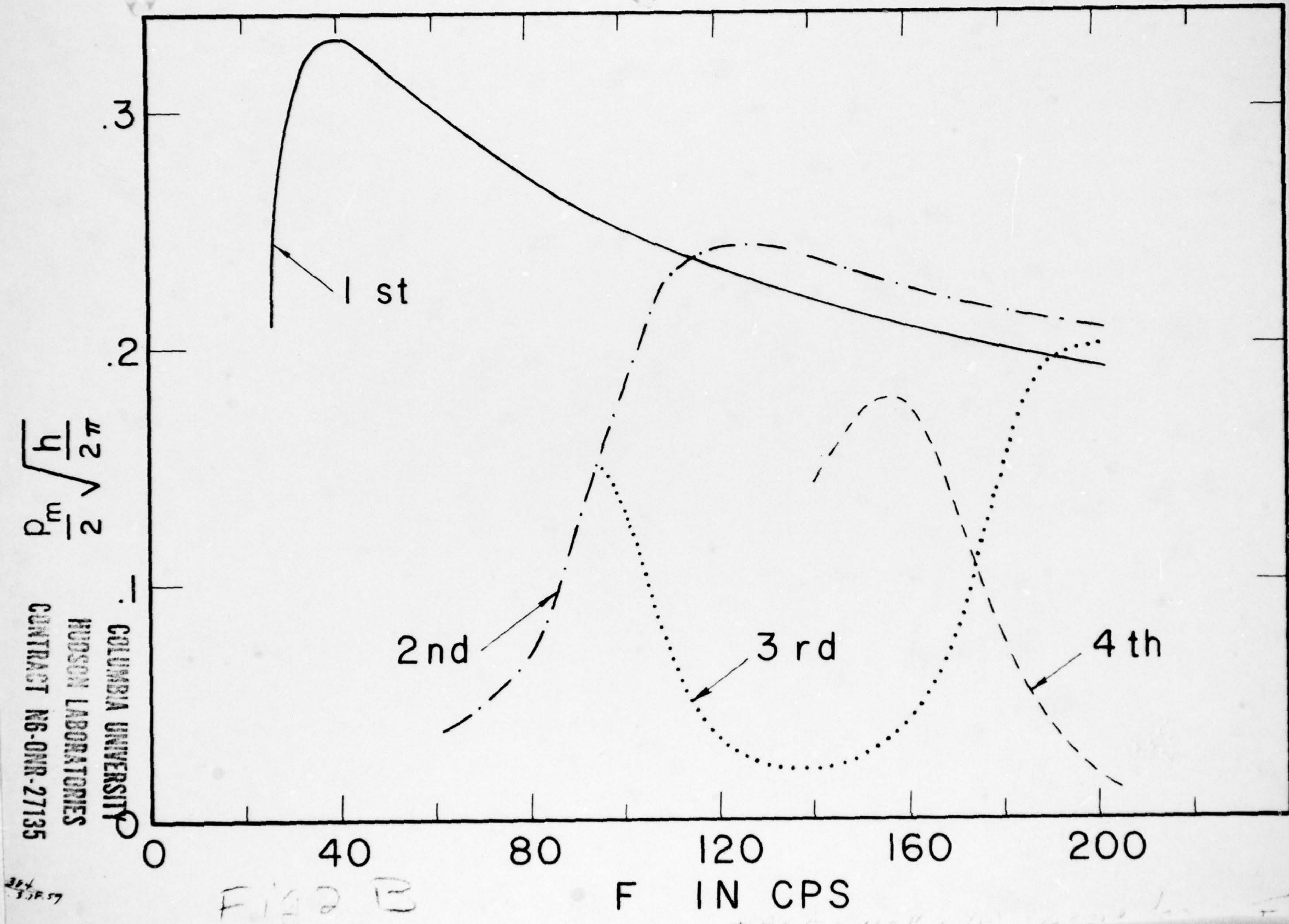

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FIG 2 B

Fq. (4.3).

The bandwidth of interest is the product of the filter function $G(\omega)$ and the mode excitations $p_m p_n$. The product may be approximated by an ideal filter with a bandwidth $\Delta\omega$ and center frequency ω_o. The terms in Eq. (4.1) can be integrated with Eqs. (4.2) and (4.3). The change of r_m over the range of $\Delta\omega$ is small enough that r_m can be considered constant for this example. For $\Delta\omega$ about $1/15\,\omega_o$, the errors are less than 10 percent of the mode interference terms.(10) The result of integration over the bandwidth $\Delta\omega_{mn}$ is the following:

(4.4)

$$F_{12} \approx \frac{A^2 \rho^2}{R_1^{1/2} R_2^{1/2}} \sum_{m,n} \bar{p}_m \bar{p}_n \sin r_{1m} z_1 \sin r_{1n} z_2 \sin r_{1m} d \sin r_{1n} d$$

$$\times \cos \beta \, \frac{\sin \Delta\omega_{mn} \delta/2}{\Delta\omega_{mn} \delta/2} ,$$

in which

$$\delta = \left(\frac{R}{U_m} - \frac{R}{U_n} \right) - \left(\frac{x_1}{U_m} - \frac{x_2}{U_n} \right) \sin \theta - (\tau_1 - \tau_2) \Big|_{\omega=\omega_o} ,$$

$$\beta = \left[(K_m - K_n) R - (K_m x_1 - K_n x_2) \sin \theta - \omega_o (\tau_1 - \tau_2) \right] \Big|_{\omega = \omega_o} ,$$

$\Delta\omega_{mn}$ = equivalent ideal filter bandwidth for the product $G(\omega) p_m p_n$,

$\bar{p}_m$ = the average value of p_m in the frequency band $\Delta\omega_{mn}$, and

$$G(\omega)p_m p_n = \bar{p}_m \bar{p}_n / \Delta\omega_{mn} \text{ for } (\omega_o - \Delta\omega_{mn}/2) \le \omega \le (\omega_o + \Delta\omega_{mn}/2)$$

or

$$G(\omega)p_m p_n = 0 \text{ for } \omega \text{ otherwise.}$$

K_m, U_m, and r_m are evaluated at $\omega = \omega_o$ in Eq. (4.4). The $n = m$ terms are the covariance of the same mode observed at the two detectors. With the range of time delay $|\tau_1 - \tau_2|$ limited to about the travel time between the detectors $|(x_1 - x_2)/U_m|$, the most important terms in the summation are the $n = m$ terms for very large source distance. The cross terms can have a contribution for time delays that are about the difference of the travel times of energy in the different modes. The time delays that maximize the covariance of the cross terms or mode interference terms are related to the distance from the source to the receiver. For a single detector, the auto-covariance function can be used to determine the source distance.

We assume in the array steering problem that the source is at great distance and that the time delay range is limited to that required to steer the array. The cross terms, $n \ne m$, will be dropped. With these assumptions and simplifications Eq. (4.4) becomes

(4.5) $$E_{12} \approx \frac{A^2 \rho^2}{R} \Sigma \bar{p}_m^2 \sin r_{1m} z_1 \sin r_{1m} z_2 \sin^2 r_{1m} d$$

$$\times \cos(\omega_o \tau - K_m x \sin\theta) \frac{\sin \frac{\Delta\omega_m}{2}\left(\tau - \frac{x \sin\theta}{U_m}\right)}{\frac{\Delta\omega_m}{2}\left(\tau - \frac{x \sin\theta}{U_m}\right)},$$

in which

$$x_1 = x/2 \quad ,$$

$$x_2 = -x/2 \quad ,$$

$$\tau = \tau_2 - \tau_1 \quad ,$$

$$R \approx R_1^{1/2} R_2^{1/2} \quad ,$$

and

$\Delta\omega_m$ = equivalent ideal filter bandwidth for the product $G(\omega)p_m^2$.

The $m\underline{th}$ term in the summation is the covariance of the acoustical pressures at the detectors for the $m\underline{th}$ mode. The size of each term is dependent upon the detector positions and source depth, in addition to the array steering parameters.

The maximum value of the contribution of the $m\underline{th}$ mode to the total covariance E_{12} is determined by the mode excitation p_m and source depth d , i.e., the value of $\bar{p}_m^2 \sin^2 r_{1m} d$.

The detector depths enter as the product $\sin r_{1m} z_1 \sin r_{1m} z_2$. The value of this product can be plus, minus, or zero. Since we are interested in the covariance of the same mode at two detectors and the total covariance as a function of steering, we assume that the detectors are at the same depth.

The dependence of the envelope of the covariance function upon steering is given by the term of form $\sin\xi/\xi$. The maximum of $\sin\xi/\xi$ is at $\xi = 0$. The value of time delay τ

for which the function is maximum is dependent upon the group velocity U_m (for $x \sin \theta$ not zero). The $\sin\xi/\xi$ term multiplies the term $\cos(\omega_o\tau - K_m x \sin \theta)$ that is also dependent upon the steering and mode. This term gives the oscillation of the covariance of the m<u>th</u> mode as a function of steering. It is dependent upon the angular frequency ω_o and the horizontal component of the wave number K_m (or in terms of the phase velocity, $C_m = K_m/\omega_o$). The phase and group velocities of energy propagation in a waveguide are generally different, and even for a single mode the steering time delay τ for maximum of the covariance may not correspond to the direction to the source. Since the group velocity U_m and phase velocity C_m are dependent upon the mode, the covariance of the acoustical pressures for each mode is a maximum at different time delays (for $x \sin \theta$ not zero). If τ is zero and $\sin \theta$ is varied, i.e., if the source is moved across the acoustic axis of the array or the array is turned for a stationary source, then all of the terms are maximum at $x \sin \theta = 0$. For either type of steering, the sum of the terms is complicated, and it is more instructive to consider specific numerical examples.

5. Numerical Example

The shallow-water area off Fire Island has been used for studies of propagation of acoustic waves in a layered waveguide. The agreement of the experimental data with theoretical calculations has been good.[10, 18] The waveguide layering and the curves of ω vs the horizontal and vertical components of wave number for the Fire Island area are shown in Figs. 1 and 2.

The source is assumed to be about 7 m deep. The detectors are assumed to be 500 m apart and 19 m deep. The center angular frequency of the source is assumed to be 300π rad/sec with bandwidth of 20π rad/sec . With these numbers, the contribution to the covariance of the 1st, 2nd, and 4th modes are nearly equal, and the 3rd mode is nearly zero. Numerical values of K_m and U_m are obtained from Fig. 2. The amplitude factors of the modes are nearly equal. The normalized covariance $\bar{E}_{12}$ is the following:

(5.1)

$$\bar{E}_{12}(\tau) = 1/3 \sum_{m=1,2,4} \cos(\omega_o\tau - K_m x \sin\theta) \frac{\sin(\Delta\omega/2)(\tau - x\sin\theta/U_m)}{(\Delta\omega/2)(\tau - x\sin\theta/U_m)} .$$

The covariance function for an array in an infinite homogeneous medium is obtained by letting the phase and group velocities be equal for all modes. For the last case, the function is of the following form:

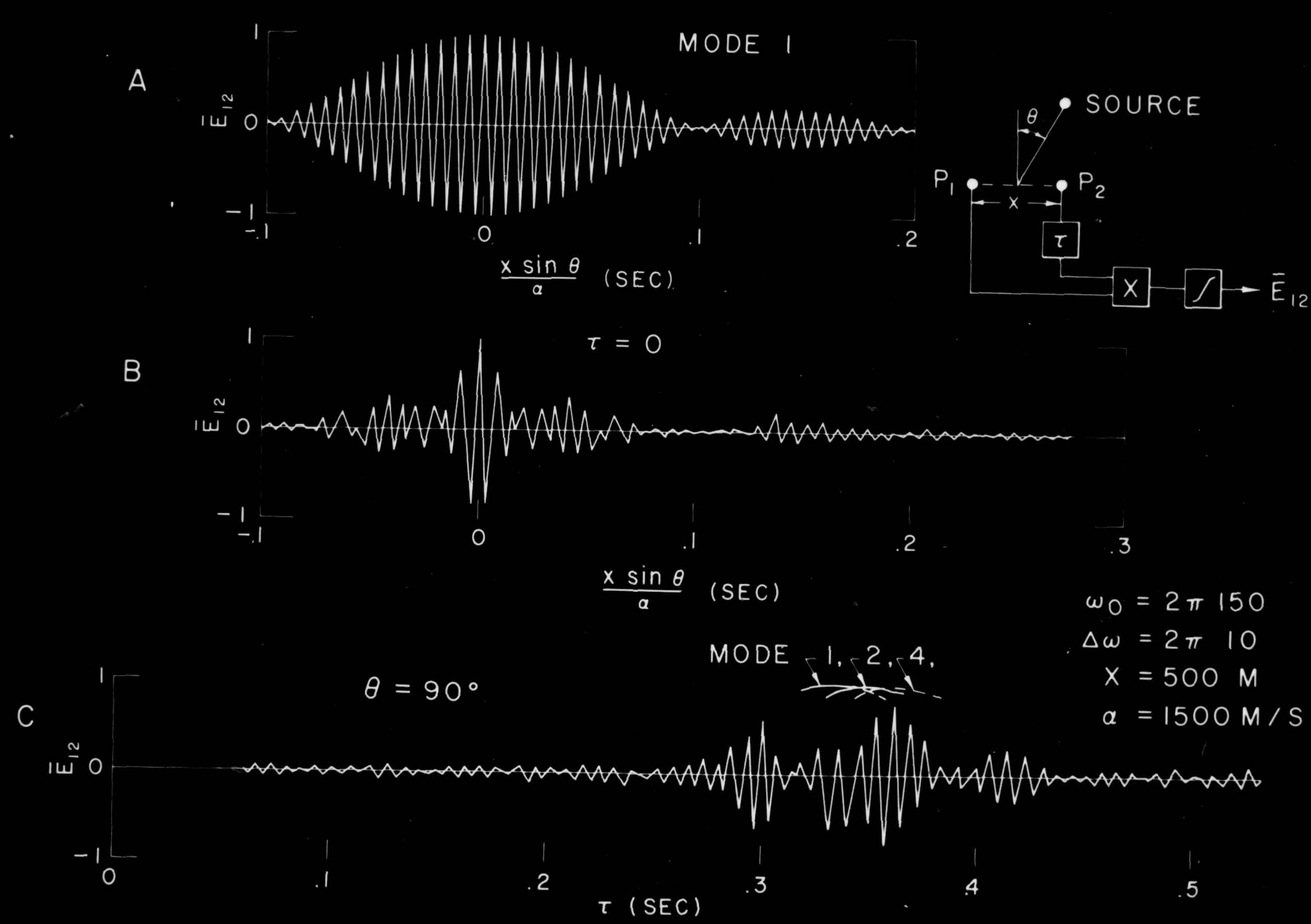
$$\cos\omega_o\xi \frac{\sin\Delta\omega\xi/2}{\Delta\omega\xi/2} ,$$

where

$$\xi = \omega_o\tau - x\sin\theta/a_1 ;$$

the covariance is the same function for either time delay or $\sin\theta$ steering.

The results of the calculations are shown in Fig. 3. Figure 3A shows the covariance of two elements in an infinite



A
MODE 1
$\bar{E}_{12}$
1
0
−1
−.1
0
.1
.2
$\frac{x \sin\theta}{\alpha}$ (SEC)
SOURCE
θ
P_1
P_2
x
τ
X
$\bar{E}_{12}$
B
$\tau = 0$
.3
C
$\theta = 90°$
MODE 1, 2, 4,
$\omega_0 = 2\pi\ 150$
$\Delta\omega = 2\pi\ 10$
X = 500 M
α = 1500 M/S
.4
.5
τ (SEC)

homogeneous ocean. This result is similar in appearance to the covariance of a single mode in the shallow-water waveguide for $\tau = 0$ and $\sin\theta$ steering. Figure 3B shows the total covariance function for $\sin\theta$ steering and three modes propagating. The sharp maximum at $x \sin\theta = 0$ and the lack of the $\sin\xi/\xi$ envelope of the total covariance function are due to the difference between the κ_m's of the different modes.

Figure 3C shows the total covariance for the second numerical example. The source is assumed to be in the direction $\theta = 90$ deg or in line with the detectors. The covariance function for each of the modes is similar in appearance to that in Fig. 3A. However, the frequency of oscillation is different for each of the modes, and the phase shifts of the cosine waves are also different. The result of this is that the covariance functions for the modes constructively and destructively interfere with each other. The phase and group velocities are not equal, and the phase of the cosine wave is not zero at a steering delay such that the $\sin\xi/\xi$ term is maximum. Since the group velocities of different modes are different, the envelopes of the covariance functions $\sin\xi_m/\xi_m$ are maximum at different values of time delay. The positions of the maxima of the envelopes for the three modes are indicated. The exact shape of the interference pattern in Fig. 3C is very much dependent upon the source direction and the detector separation $x \sin\theta$.

If we were to compute the total covariance for a small range of θ near θ_o, we would find that the interference maxima and minima would describe an envelope that is approximately of the form $\sin\xi/\xi$. This suggests that the total covariance function could be squared and averaged over a small range of $x \sin \theta$. (This type of data processing does not seem to be advantageous because it requires many determinations of the covariance squared as $x \sin \theta$ is varied.) It may be possible to combine the outputs of several detectors at different depths so as to detect each mode separately. The separate covariance function for each mode could be examined separately or recombined with suitable time delays to give a total covariance similar to that in Fig. 3B.

6. Conclusions

Steering an array in a waveguide is more difficult than steering an array in an infinite homogeneous medium. The reason is that the wave propagation in a waveguide is more complex. The energy propagates in many modes, and each mode can have different phase and group velocities. Although we have not studied the signal-to-incoherent-noise problem, it is reasonable to compare signal-to-noise studies with the array steering problem. In both cases we assume that we are measuring the covariance of the acoustical pressure observed at two detectors. A convenient definition of the output signal-to-noise ratio of a covariance detector is the ratio of the covariance squared to the mean square fluctuations of the covariance.[2] In this respect then, fluctuations of the covariance function for very small changes of source direction can be considered as an apparent noise.

Mechanical or sin θ steering gives a covariance of the acoustical pressures with a well-defined maximum. The covariance maximum is in the proper direction. The width of the maximum of the envelope of the covariance is narrower than would be expected on the source bandwidth. This might be expected since several independent pieces of information, i.e., several propagation modes, are combined in this calculation. The maximum of the covariance function should not fluctuate for small changes of the source direction. In the absence of incoherent background noise, the apparent noise level for this type of array steering should be very low.

Time delay steering of a two-element array that detects all modes yields a covariance that has several maxima and minima. The complication is caused by the constructive and destructive interference of the covariance functions of the several modes. The magnitude of the interference maxima and minima is less than or equal to the sum of the envelopes of the covariance functions for each mode. The maximum of the envelope of the covariance of each mode occurs at a time delay that is determined by the source direction, detector separation, and group velocity of the mode. I suggest that it may be necessary to average the square of many covariance measurements to determine the source direction with time delay steering. Even so, the sum of the different maxima of the envelopes for the different directions would broaden the covariance function. The fluctuations of the covariance may be large in this case and correspondingly the apparent noise level large.

This study has been restricted to a noise source in a waveguide with infinite layers. The layers are assumed to have constant thickness and constant properties. If the layers are not uniform in the area of the array, the acoustical pressures at the two detectors $P_1(t)$ and $P_2(t)$ can be different. The non-uniform conditions can cause the covariance function to be small and erratic. Under these conditions, measurement of the covariance function for any type of steering would indicate that the apparent noise level is high.

As a final consideration let us assume that we have a simple harmonic source in the waveguide. The frequency of the acoustical pressure is the same for all detector positions regardless of any non-uniform conditions or the number of modes propagating. This means that the covariance of the acoustical pressures is a function of the phase difference of the acoustical pressures and that the covariance may be large, whereas under the same conditions with a noise source having small bandwidth, the covariance would tend to zero. For these reasons it is difficult to predict the performance of a more complex array in an irregular waveguide on the basis of studies made with a harmonic source and a two-element array.

Acknowledgments

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